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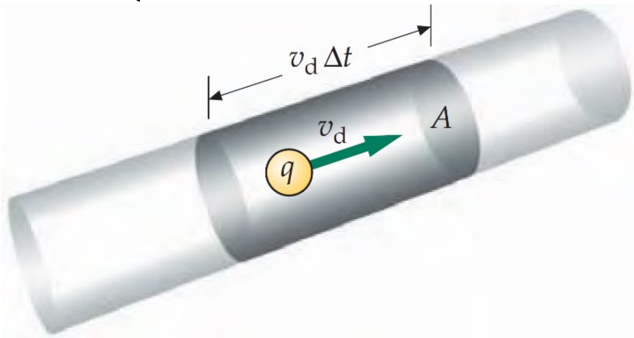
目录

Current and the motion of Charges 电流与电荷运动

初等数学	高等数学
$I=\frac{\Delta Q}{\Delta t}$	$I=\lim_{\Delta t\to 0}\frac{\Delta Q}{\Delta t}=\frac{dQ}{dt}$

$\Delta Q=qnAv_d\Delta t$

$$I=\frac{\Delta Q}{\Delta t}=qnAv_d\quad\begin{cases}n: Number\ Density\ of\ particles\\ q: charge\ of\ each\ particle\\ v_d: drift\ speed\ (\text{电荷的漂移速度})\end{cases}$$



The current density vector(电流密度矢量), $\vec{J}$, is specified by

$\vec{J}=qn\vec{v}_d$

If $\vec{J}$ is uniform and if the surface is flat, which means that $\hat{n}$ would be uniform, then the flux can be expressed

$$I=\int_S \vec{J}\cdot d\vec{A}=\vec{J}\cdot\vec{A}=JA\cos\theta$$

Example: In a certain particle accelerator, a current of 0.5mA is carried by a 5.0 MeV proton beam that has a radius equal to 1.5mm. The mass of a proton is $m=1.67\times10^{-27}kg$. The charge of a proton is $q=1.60\times10^{-19}\ C$.

注意：此处出现了新的能量单位 eV(电子伏特). 1 eV 是 1 个电子经过了 1V 的电势差后，所获得的能量。

(a) Find the number density of protons in the beam.

$$E_k=\frac{1}{2}mv^2\ \Rightarrow v=\sqrt{\frac{2E_k}{m}}$$

$$I=qnAv\ \Rightarrow n=\frac{I}{qAv}=\frac{I}{qA}\sqrt{\frac{m}{2E_k}}$$

$$=\frac{0.5}{(1.60\times10^{-19})\pi(1.5\times10^{-3})^2}\sqrt{\frac{1.67\times10^{-27}}{2(5.0\times10^{-6})}}=1.4\times10^{13}\ (\frac{protons}{m^3})$$

(b) If the beam hits a target, how many protons hit the target in 1.0s?

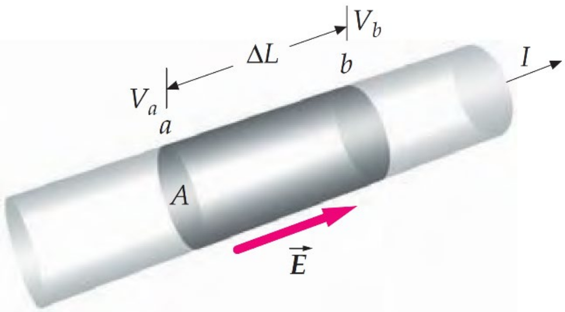
Total number $N=\frac{\Delta Q}{q}$, $I=\frac{\Delta Q}{\Delta t}$

$$\Rightarrow N=\frac{I\Delta t}{q}=\frac{0.5\times10^{-3}}{1.67\times10^{-27}}=3.1\times10^{15}\ (protons)$$

Resistance and Ohm's Law 电阻及欧姆定律

A segment of wire that has a current I. The potential drop $V_b - V_a$ is related to the electric field by:

$$V = V_b - V_a = E\Delta L$$



Resistance: the ratio of the potential drop.

$$R = \frac{V}{I}$$

SI unit: ohm (Ω)

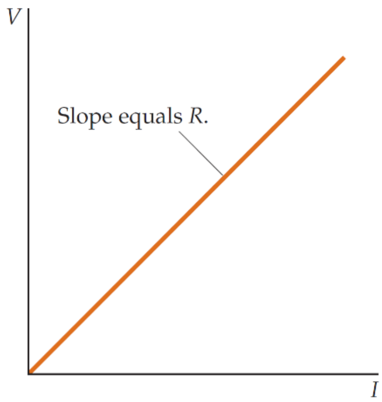
$$1\Omega = 1V/A$$

Ohmic Materials: Resistance does not depend on V or I

⇒Fixed resistance material (in a wide range of conditions)

Ohm's Law:

$$V = IR$$

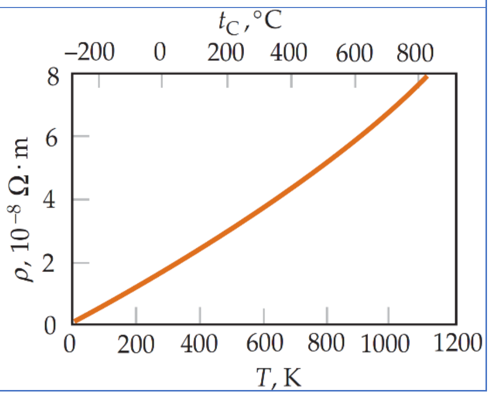


Resistance:

$$R = \rho\frac{L}{A}$$

(ρ: *resistivity*, L: *length*, A: *cross − sectional area*)

Resistivity ρ: depends on temperature



Example: Calculate the resistance per unit length of a 14-gauge copper wire: $\rho = 1.7 \times 10^{-8}$ (Ω/m), diameter $d = 1.628\ (mm)$ Area $A = 2.081\ (mm^2)$.

$$R = \rho\frac{L}{A} = 1.7 \times 10^{-8}\frac{1}{2.081 \times 10^{-6}} = 8.2 \times 10^{-3}\ (\Omega)$$

Energy in Electric Circuits 电路中的能量

$$-E_p = \Delta Q(V_a - V_b) = \Delta QV$$

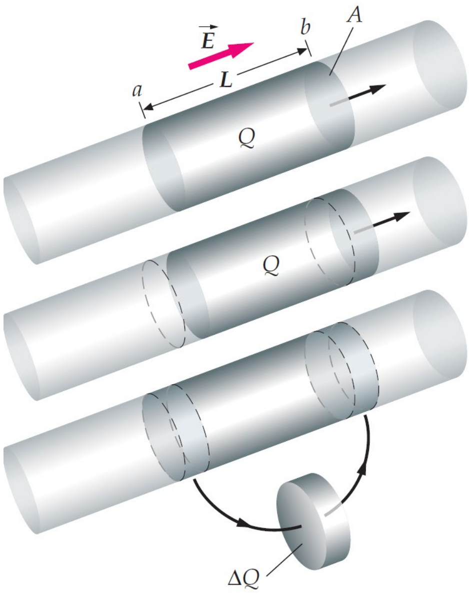
$$-\frac{dE_p}{dt} = \frac{dQ}{dt}V = IV$$

Rate of Potential Energy Loss

(Power 功率):

$$\Rightarrow P = IV$$

$$P = IV = I^2R = V^2/R$$



Example: The rated current of an ear-piece is 10mA and the equivalent resistance of it is 50 Ω. Determine the working power and rated voltage of it.

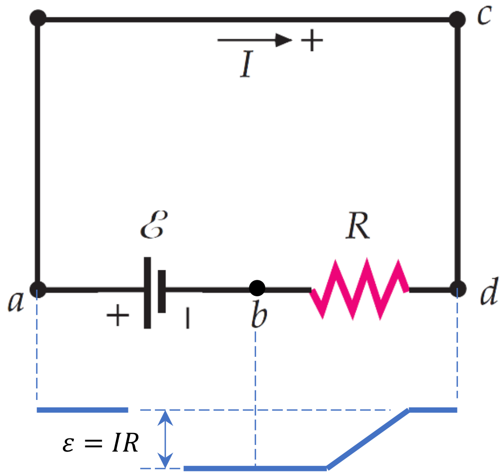
$$P = I^2R = 0.01^2 \times 50 = 5 \times 10^{-3}\ (W) = 5\ (mW)$$

$$P = IV\ \Rightarrow V = P/I\ = (5 \times 10^{-3})/(10 \times 10^{-3}) = 0.5\ (V)$$

Battery 电池:

An ideal battery is a source of energy that maintains a constant potential difference between its two terminals (independent of current through the battery).

The work per unit charge it provided is called ε(electro-motive force 电动势)



$$P = \frac{\Delta Q}{\Delta t}\varepsilon = I\varepsilon$$

(功率 = 电流 · 电动势)

$$V_a - V_b = IR = \varepsilon \Rightarrow I = \frac{\varepsilon}{R}$$

$E = Q\varepsilon$ (能量 = 电荷量 · 电动势) $I = \Delta Q/\Delta t$

A real battery consists of an ideal battery and an *internal resistance r*.

$$V_a - V_b = IR = \varepsilon - Ir$$

$\Rightarrow I = \frac{\varepsilon}{R+r}$ 或 $\varepsilon = Ir + IR$

Example: A small bulb of 11Ω is connected across a battery of 36 V and internal resistance 1Ω. Find

a. the current

$$I = \frac{\varepsilon}{R+r} = \frac{36}{11+1} = 3\ (A)$$

b. the terminal voltage of the battery

$$V_a - V_b = IR = 3 \times 11 = 33\ (V)$$

c. the power supply by the chemical reactions

$$P = I\varepsilon = 3 \times 36 = 108\ (W)$$

d. the power delivered to the bulb.

$$P = I^2R = 3^2 \times 11 = 99\ (W)$$

e. the power consumed by the internal resistance.

$$P = I^2r = 3^2 \times 1 = 9\ (W)$$

f. If the battery is rated at 15 A · h, how much energy does the battery store?

$$1\ h = 3600\ s$$

$$E = Q\varepsilon = (I\Delta t)\varepsilon = 15 \times 3600 \times 6 = 3240\ (J)$$

思考下面这样计算能量为什么不对?

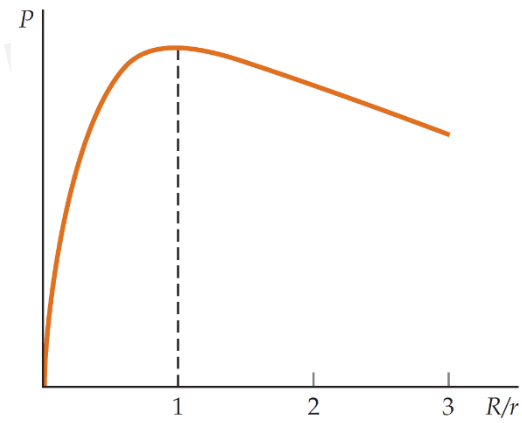
$$E = Pt = 108 \times 3600 = \cdots$$

Example: For a battery that has an emf equal to ε and internal resistance equal to r what value of external resistance R should be placed across the terminals to obtain the maximum power delivered to the resistor?

$$I = \frac{\varepsilon}{R+r}$$

$$P = I^2R = \frac{\varepsilon^2 R}{(R+r)^2}$$

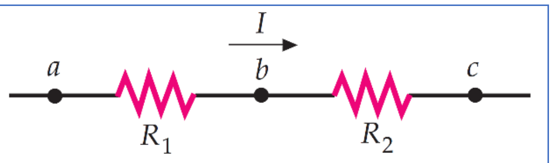
$$\frac{dP}{dR} = 0 \Rightarrow R = r$$



Resisters in Series(串联电阻)

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

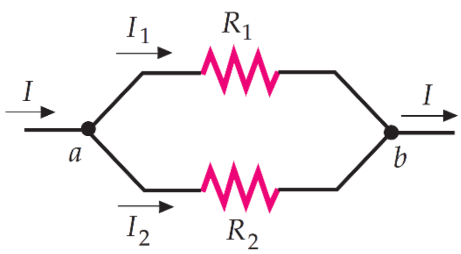
- 串联**电路中**电流 I 处处相等
- 串联电路总电压是各个分电压之和



Resisters in Parallel(并联电阻)

$$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \cdots$$

- 并联**电路两端**电压 V 相等
- 并联电路总电流是各个分电流之和



Kirchhoff's Rules 基尔霍夫定律

1 The node rule: $I_{in} = I_{out}$
2 The loop rule: closed loop ⇒ Potential Difference be 0

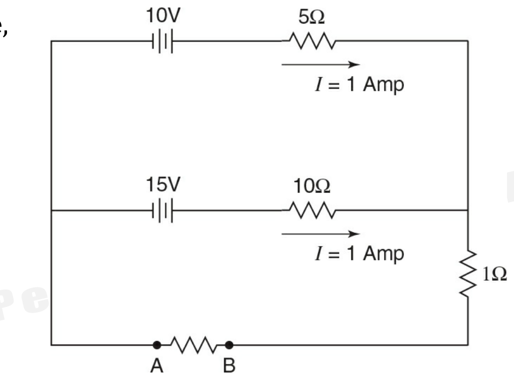
解题步骤：

1. 找到 node
2. 找到 loop

解题关键:

1 正负号：电池正负极不要搞错。
2 正负号：电流方向不要搞错，沿电流方向电压升高还是降低不要搞错。

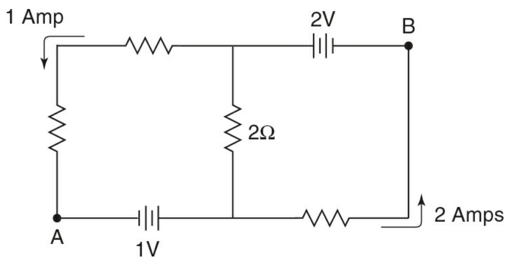
Example For the circuit in the right figure, which of the following is true?



(A) $V_B - V_A = -2V$
(B) $V_B - V_A = +3V$
(C) $V_B - V_A = -3V$
(D) $V_B - V_A = -5V$
(E) $V_B - V_A = +5V$

Barron 14-5 答案(B)

Example In the right, if the potential at point A is chosen to be zero, what is the potential at point B?



(A) +1V
(B) -1V
(C) +2V
(D) -2V
(E) +3V
Barron 14-10 答案(A)

Capacitance and RC Circuit 电容及 RC 电路

- Capacitance: $C = Q/V$
  Isolated spherical conductor $C = 4\pi\varepsilon_0 R$
  Parallel-plate capacitor $C = \varepsilon_0 A/d$
- Energy in Capacitors: $dU = VdQ = QdQ/C \Rightarrow U = \int \frac{Q}{C}dQ = \frac{Q^2}{2C}$

  $U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$
- Dielectrics: $C = kC_0 \quad \varepsilon = k\varepsilon_0$
  $\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$
- Equivalent Capacitance:
  Parallel capacitors $C_{eq} = C_1 + C_2 + C_3 + \cdots$
  Series capacitors $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \cdots$

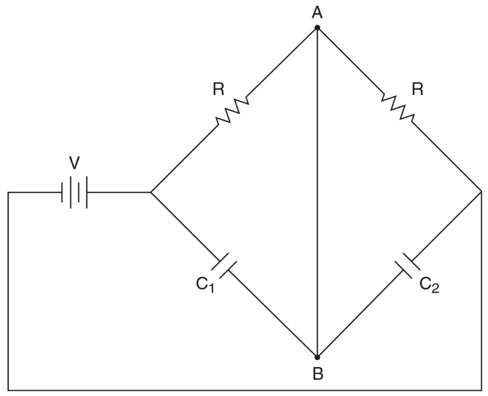
对比

电容：$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$

电阻：$P = IV = \frac{V^2}{R} = I^2R$

Example When the circuit shown in the right figure has reached a steady state, what is the charge on capacitor 1 (which has a capacitance C1, as shown)?

(A) $C_1^2V/(C_1 + C_2)$
(B) $C_1C_2V/(C_1 + C_2)$
(C) $C_2^2V/(C_1 + C_2)$
(D) C_1V
(E) $\frac{1}{2}C_1V$



Barron 15-9 答案(E)

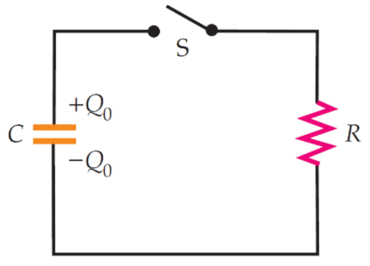
Discharging a capacitor through a resistor

$$\begin{cases} V_C = V_R \\ I(t) = -\frac{dQ}{dt} \\ I_0 = \frac{V_0}{R} = \frac{Q_0}{RC} \end{cases}$$

$$V_C = V_R \Rightarrow \frac{Q}{C} = IR = -\frac{dQ}{dt}R$$

$$Q(t) = Q_0 e^{-\frac{t}{RC}}$$

$$I(t) = -\frac{dQ}{dt} = \frac{Q_0}{RC}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$



(b)

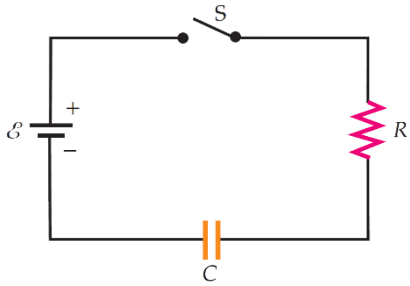
Charging a capacitor

$$\begin{cases} \mathcal{E} = V_R + V_C = IR + Q/C \\ I(t) = +\frac{dQ}{dt} \\ I_0 = \frac{\mathcal{E}}{R} = \frac{Q_0}{RC} \end{cases}$$

$$\mathcal{E} = IR + \frac{Q}{C} = \frac{dQ}{dt}R + \frac{Q}{C}$$

$$Q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right) \quad Q_{final} = C\mathcal{E}$$

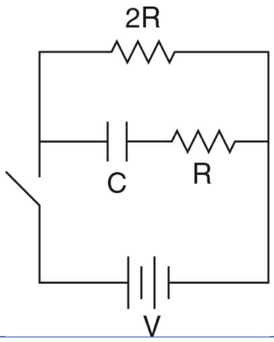
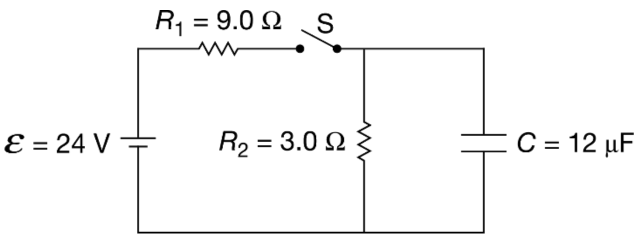
$$I(t) = I_0 e^{-\frac{t}{RC}}$$



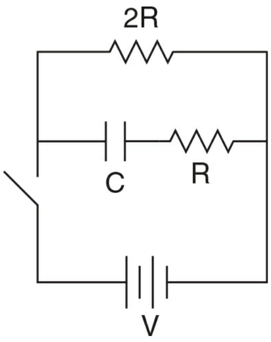
Time constant

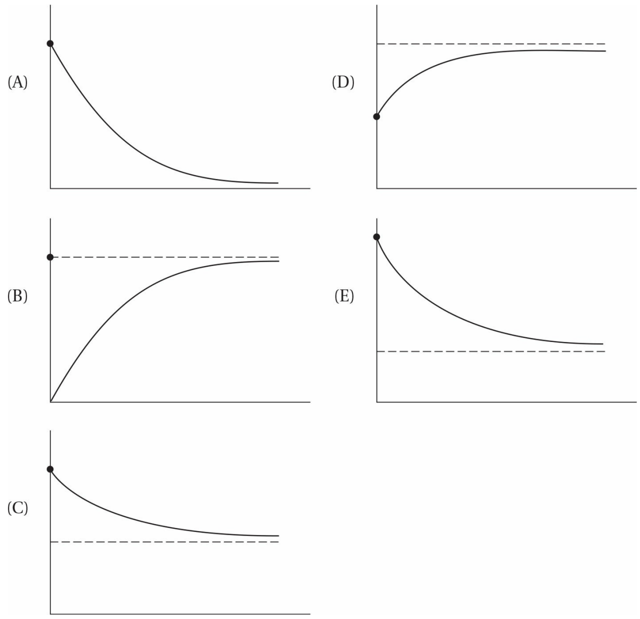
$\tau = RC$ (second)

图示电路在开关闭合时充电，开关断开时放电。分析在充电和放电时的时间常数,初始电流。

Barron 16-9	寒假作业 Question 24
	
Charging: $\tau = RC$ Discharging: $\tau = 3RC$	Charging: $\tau = (9\Omega)(12\mu F) = 108\ (\mu s)$ Discharging: $\tau = (3\Omega)(12\mu F) = 36\ (\mu s)$
Charging: $I_0 = \frac{V}{R}$ Discharging: $I_0 = \frac{V}{3R}$	Charging: $I_0 = \frac{\mathcal{E}}{R} = \frac{24}{9} = \frac{8}{3}\ (A)$ Discharging: $I_0 = \frac{V}{R} = \frac{6}{3} = 2\ (A)$

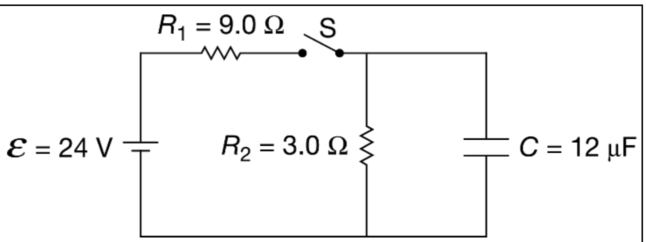
Barron 16-9: For the circuit in the right figure, which of the graphs in Figure 16.7 shows the current through the battery as a function of time?





答案(E)

寒假作业 Question 24



A power supply is set to $\mathcal{E}$ =24V and is connected to resistors R_1=9.0Ω and R_2=3.0Ω , capacitor C=12μF , and switch S , as shown in the figure. Initially, the capacitor is uncharged, and switch S is open.

(a) At time t=0 , the switch is then closed.

i. Calculate the current through R_1 immediately after the switch is closed.

After the switch is closed, the capacitor branch is short.

The current through R_1 is I_1 : $I_1 = \mathcal{E} / R_1 = 24/9 = 8/3$ (A)

ii. Determine the current through R_2 immediately after the switch is closed.

As the capacitor branch is short, the voltage across C and R_2 is equal 0.

The current through R_2 is I_2 :$I_2 = 0/3.0 = 0$ (A)

A long time after the switch is closed, the circuit reaches steady-state conditions.

(b) Calculate the potential difference across R_2 .

A long time after the switch is closed, the circuit reaches steady-state conditions so that the current through C is 0 and the voltage across C equals to the potential difference across R_2.

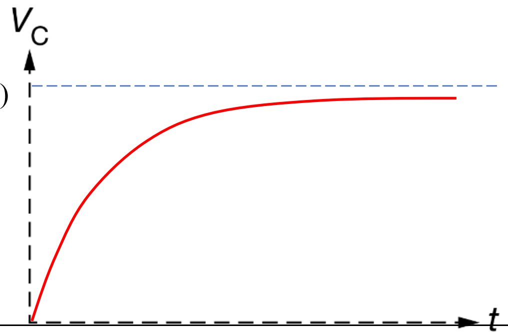
$R_{12} = R_1 + R_2 = 12$ (Ω)

The voltage across R_2 is V_2 : $V_2 / \mathcal{E} = R_2 / R_{12}$ $V_2 = 6$ (V)

(c) Calculate the magnitude of the charge Q on the positive plate of the capacitor.

Q=V*C=6*12=72 (μC)

(d) On the axes shown, sketch a graph of the potential difference V_C across the capacitor as a function of time t . Explicitly label any intercepts, asymptotes, maxima, or minima with values or expressions, as appropriate.



The voltage at time t=0 is V(0)=0 (V)
The voltage at time t=∞ is V(∞)=6 (V)

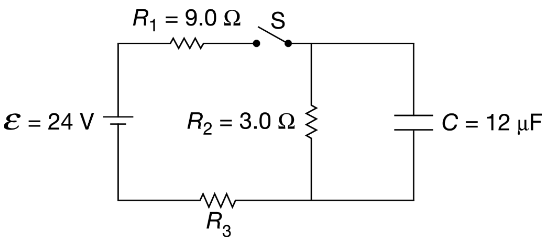
After steady-state conditions are reached, the switch is now opened, and time is reset to t=0.

(e) Using integral calculus, derive an expression for the charge q(t) on the capacitor as a function of time t after the switch is opened. Express your answer in terms of Q.

After the switch is opened, the capacitor will discharge through resister R_2.

$q(t) = Q_0 e^{\frac{-t}{RC}}$ where $Q_0 = 72\ \mu C$, C=12 μF, R= R_2=3.0 Ω.

The capacitor is discharged, and a third resistor is added to the circuit, as shown above. The switch is then closed.



(f) Does the time it takes for the charge on the capacitor to reach 2/3 of its maximum value increase, decrease, or stay the same as compared to the circuit in part (a)?

____Increase ____Decrease ____Stay the same

Justify your answer.

As total resistance is increased after the new resistor is inserted, according to the charging equation:

$$Q(t) = Q_0 e^{\frac{-t}{RC}}$$

Where R is increased. So that the time it takes for the charge on the capacitor to reach 2/3 of its maximum value would increase.

__■__Increase